كلية الهندسة بشبرا
FACULTY OF ENGINEERING- SHOUBRA

Electromagnetic Fields

Part 2 - Lecture (3)

Magnetic Fields Part I

Assoc. Prof. Dr/ Moataz M. Elsherbini
motaz.ali@feng.bu.edu.eg

Chapter Contents

$\boxed{1}$ Static Magnetic Field (Magnetostatics)

Notes:- • Any wave in air composed of 2 components (E & H)

• This waves carries a message to transmit it betⁿ Tx & Rx

• $E \perp H$ in x, y, z coordinates & so the third direction will be the propagation direction, so Magnetic field is important to be studied.

أى موجه ماشيه فى الهواء مكونة من E ، H متعامدين وحاملة معلومة ينبغى توصيلها للمكان لذا لزم دراسة H مثل E.

(a)

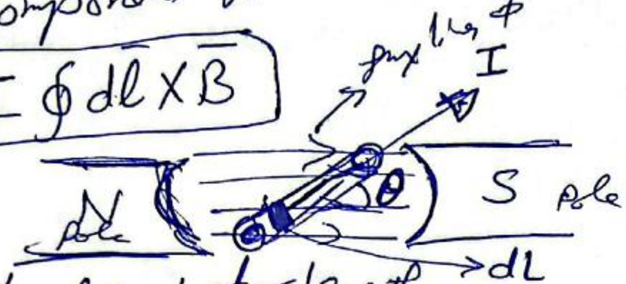
- <u>Study a moving charge or "charged wire"</u> سلك مشحون

→ If there is a moving charge (wire), So a magnetic Field Generated around the wire. & The direction of this Field depends on "R.H.S" Rule

لو عندي سلك فيه شحنات تتحرك ينتج تيار ومنه ينتج مجال مغناطيسي ، صابع الإبهام يشير لـ current ، دوران اليد يشير لحركة H وتنتج المجال في صورة دوائر متحدة المركز ، مركزها السلك المار فيه تيار ، تسبب في وجود H .

(b) If a wire-charged & carry current I, lies between Two Poles (any Two Poles, generate Magnetic flux between them (L.H.R)) we Notice that the wire Moves because there is a Force generated (Lorentz Force) Make the wire moves.

- Now if the wire conductor inclined by angle θ with the magnetic flux, so the normal component will be only effective (active) $\therefore$ $\boxed{\vec{F} = I \oint \overline{dl} \times \overline{B}}$



$$\boxed{|\vec{F}| = BIdl \sin\theta}$$

- where $\overline{dl}$ is differential element of conductor length

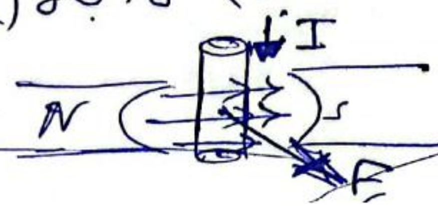
- If wire was $\perp$ flux line $\therefore$ $F = BIdl$ | B.. magnetic flux density, [illegible] Electricity I |
- " " "$\parallel$" to " " $\therefore$ $F = 0$

(L.H.R)

* We use "Felemeng" Rule to determine Force direction

الإبهام ناحية (Force) و السبابة (Field) و الوسطى (Current)

[illegible] (L.H) [illegible] outward [illegible] F ال →



2- BIOT-SAVART'S RULE

$\boxed{2}$ Biot - Savart

تيار - زمن - طول

$\therefore |\vec{F}_{magnetic}| = q\vec{v} \times \vec{B} = qvB\sin\theta$ (v = velocity)

$\because I = \frac{dq}{dt}$ or $\approx \frac{q}{time}$, $q = It$, $v = \frac{l}{t}$

$\rightarrow |\vec{F}| = I * t * v * B\sin\theta = I * \cancel{t} * \frac{l}{\cancel{t}} * B\sin\theta$

$$\therefore |\vec{f}| = BIl\sin\theta$$

نظرية Biot Savart تعمم على حساب Magnetic Flux عند قطب Pole

على الحواف

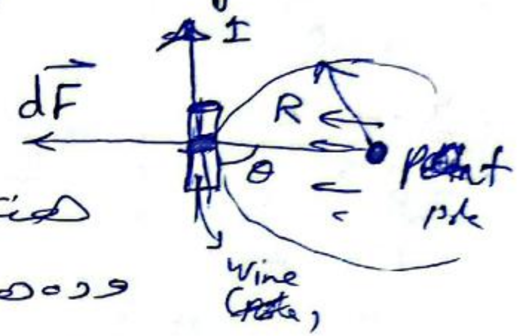
[3] Biot-savart :

1- * We want to calculate Magnetic flux (field) at point "p" due to "Pole". the point

2- * Imagine a "sphere" of radius "r" & a point lies on the surface of sphere.

* assume The Pole lies inside the Center of sphere with $\phi = 1 wb$ (١ ويبر)

* The flux (1wb) radiated From the Center of sphere To all directions Passes through sphere surface



باختصار عندي سلك عرضي فيه تيار ، وحنا نتخيل نقطة هي
جا ٥ من pole مطلوب احسب الـ magnetic flux عند النقطة
هنتخيل ان النقطة دي منها pole وطالع منه $1 wb = E - flx$
ودة هيسبب magnetic field في شكل دوائر متحدة المركز حول السلك ويسبب
مرور تيار .. هنا هنجزء من السلك طوله dl ونرسم كرة كبيرة تقع في محيطها
الـ pole ويبعد مسافة R عن السلك ∴ لازم يكون عمودي force

$\therefore \quad dF = B I dL \sin\theta$

Force Must be ⊥ surface و dl has angle θ

$\therefore \; dF = BIdL\sin\theta$ Force must be ⊥ surface & dl has angle θ

$\therefore \; B = \frac{\phi}{A}$ = Flux density

ϕ choosed as 1 wb & $A_{sphere} = 4\pi R^2$

$\therefore \; dF = \frac{1}{4\pi R^2} \cdot IdL\sin\theta$ $\therefore \; F = \int$ ——

$\therefore \; \boxed{F = \frac{I l \sin\theta}{4\pi R^2}}$

$\because E = F/q$

$H = F/\phi$ or $dH = \frac{dF}{\phi}$

$\phi = 1wb$

$dH = \frac{IL\sin\theta}{4\pi R^2}$

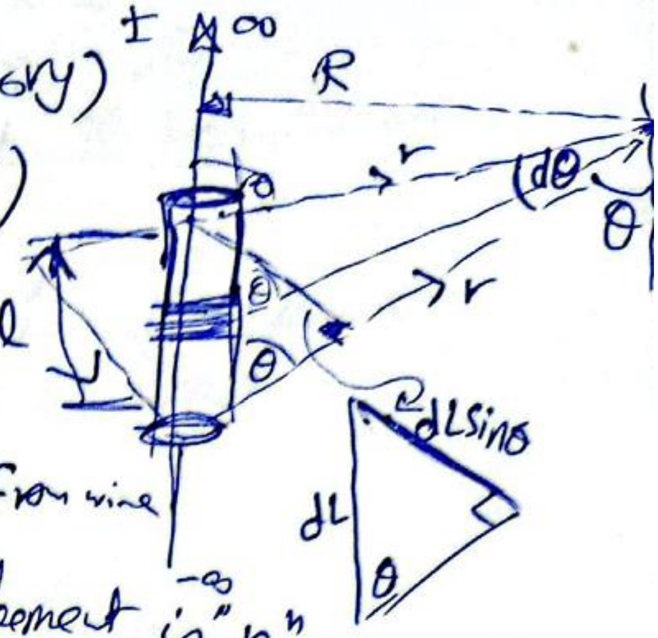
$\therefore \; \boxed{B = \mu H}$

$\therefore \; \boxed{dB = \mu dH = \frac{\mu I L \sin\theta}{4\pi R^2}}$

Biot - Savart Rule

3- AMPERE'S LAW & MAXWELL 3RD EQUATION

3 Ampere's law (dipole theory)



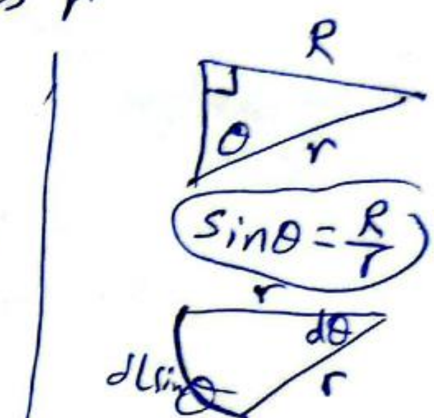
- assume very long wire (charged with I)
- choose a differential length (dL)
- measure magnetic field at "p" very far from wire ∴ with distance "R" from wire
- distance betⁿ point & differential element is "r"
- The angle between lines "dθ"

Start From Biot Savart

الـ r بين dL و [illegible]

$$dB = \frac{\mu I dL \sin\theta}{4\pi r^2}$$

$$B = \frac{\mu I}{4\pi}\int \frac{dL \sin\theta}{r^2}$$



$\sin\theta = \frac{R}{r}$

arc $= r\,d\theta$
$= dL \sin\theta$

$$B = \frac{\mu I}{4\pi} \int \frac{dL \sin\theta}{r^2}$$

$\because$ $dl \sin\theta$ like a small arc $\approx r\,d\theta$ $\therefore$

$$\therefore \boxed{dL \sin\theta = r\,d\theta}$$

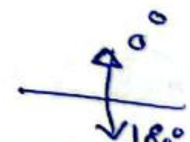
$$\therefore B = \frac{\mu I}{4\pi} \int \frac{\cancel{r}\,d\theta}{r^{\cancel{2}}} = \frac{\mu I}{4\pi} \int \frac{d\theta}{r}$$

$$\because \sin\theta = \frac{R}{r}$$ → المستقيم العمودي من P على السلك

$$\therefore r = \frac{R}{\sin\theta}$$

$$\therefore B = \frac{\mu I}{4\pi} \int \frac{\sin\theta\, d\theta}{R}$$

why $\int_0^{\pi}$



السلك لو امتد للأنهاية من ناحيتها
و امتد للأعلى
يبقى رسم لنص دائرة
مجموعها ١٨٠°

$$\therefore \boxed{B = \frac{\mu I}{4\pi R} \int_0^{\pi} \sin\theta\, d\theta}$$

$$\therefore \boxed{B = \frac{-\mu I}{4\pi R} \cos\theta \Big|_0^{\pi} = \frac{\mu I}{2\pi R}}$$

$$\because B = \mu H \quad \therefore \boxed{H = \frac{I}{2\pi R}}$$

$\boxed{5}$ ∴ $H = \frac{I}{2\pi R}$ ∴ $H * 2\pi R = I$

$H *$ محيط $=$ تيار

∴ $\boxed{\oint_c H \cdot dL = I_{en}}$ If we Integrate all points on the wire So $\oint_c H \cdot dL$ is Magnetic field along all circumference gives Total current enclosed

تشبه معادلة جاوس

Magnetic تكتب أن $\boxed{\text{Gauss}}$

in Electrical

$\oint \bar{E} \cdot d\bar{S} = \frac{Q_{en}}{\epsilon_0}$

in Magnetic

$\oint \bar{H} \cdot d\bar{l} = I_{en}$

For any wire with thickness I replaced with J

∴ $\boxed{\oint \bar{H} \cdot d\bar{L} = I = \iint_s \bar{J_s} \cdot d\bar{s}}$ Ampere' law Integral Form

Now using $\boxed{\text{Stock theorem}}$

$$\oint \bar{H} \cdot d\bar{l} = \iint (\nabla \times \bar{H}) \cdot d\bar{s} = \iint \bar{J_s} \cdot d\bar{s}$$

Line Integral surface Integral

∴ $\boxed{\nabla \times \bar{H} = \bar{J_s}}$ معادلة ماكسويل الثالثة Maxwell's 3rd equ

* ومضيف عليها ماكسويل جزء $\frac{\partial D}{\partial t}$ في حالة Time varying field

4 – GAUSS IN MAGNETIC FIELD & 4^{TH} MAXWELL EQUATION

* Magnetic Gauss

The Magnetic flux density over any closed surface $= 0$

Integral form $\rightarrow$ $\oint\!\!\oint \overline{B} \cdot \overline{ds} = \iiint (\nabla \cdot \overline{B})\, dv = 0$ $\leftarrow$ using divergence theorem

$= \boxed{\nabla \cdot \overline{B} = 0}$ differential form

تكوين تكامل على سطح مغلق

لا ج. معادلات ماكسويل

$\rightarrow$ 4Th eqn of Maxwell's

5- ALL MAXWELL EQUATIONS IN STATIC FIELD

Now All Maxwell's eqn
For static E & M Fields

Differential Form		Integral Form
(1) $\nabla\cdot\bar{D}=\rho_v$	Gauss Electric	$\oint_s \bar{D}\cdot\bar{ds}=\iiint \rho_v\, dv$
(2) $\nabla\cdot\bar{B}=0$	Gauss Magnetic	$\oint_s \bar{B}\cdot\bar{ds}=0$
(3) $\nabla\times\bar{E}=0 + \frac{\partial B}{\partial t}$	Farady Elec.	$\oint_c \bar{E}\cdot\bar{dl}=0$
(4) $\nabla\times\bar{H}=\bar{J}+\frac{\partial D}{\partial t}$	Ampere's Magnetic	$\oint_c \bar{H}\cdot\bar{dl}=\iint_s \bar{J}\cdot\bar{ds} = I_{en}$

6 - SOLENOID

4 Solenoid

- If you have an iron core (قلب حديد), and a wire winded around it ملفوف حوله السلك (قلب حديد core) with carrying current (I), ∴ a magnetic field produced in circular form تطلع مجال مغناطيسي على شكل دوائر

∴ $\oint_c \bar{H} \cdot \bar{dL} = I$, For no. of turns ($N$) ∴ total current is $N \cdot I$

$$\therefore \oint \bar{H} \cdot dL = NI \quad \therefore H = \frac{NI}{L}$$

$$\therefore H \cdot L = NI$$

$$\therefore H = \frac{NI}{L}$$

لو سحبت القلب الحديد يتبقى عندي ملف له حث

Not: If I withdraw the core from between the winding so I got a coil with Inductance

If we withdraw core from winding of solenoid

∴ we have $\boxed{\text{Inductor}}$

$$\underset{\text{henry}}{L} \rightarrow \text{inductance} = \frac{N\phi}{I} \quad \& \quad \phi = BA \ \& \ B = \mu H.$$

$$\& \ H = \frac{NI}{\ell}$$

$$\therefore L = \frac{N * [BA]}{I} = \frac{NA[\overset{B}{\mu H}]}{I} = \frac{NA\mu \cdot \left(\frac{N\cancel{I}}{\ell}\right)}{\cancel{I}}$$

$$\therefore L = \mu \frac{N^2 A}{\ell}$$

ℓ → length of solenoid

معادلة مهمه لتصميم أي الملف يدوي

بأي Inductor احتاجه في دوائر الاتصالات

زي دوائر FM

TOROID

Toroid

الملفوف لولبا / دائرة مغلقة

- If we re-shape solenoid to be as a circle
- cross-sectional area of Toroid is πr^2

  مقطع الـ core

- Magnetic field will circulate on a circular path inside core only

  يعني المجال المغناطيسي داخل القلب كبير فقط
  خارجه صفر

$$L = \frac{\mu N^2 A}{l}$$

$A \rightarrow \pi r^2$ (cross sectional area of core)

$l \rightarrow$ length of circular solenoid

circumference $= 2\pi R$

$$\therefore \quad L = \frac{\mu N^2 \pi r^2}{2\pi R} \Rightarrow \boxed{L = \frac{\mu N^2 r^2}{2R}}$$

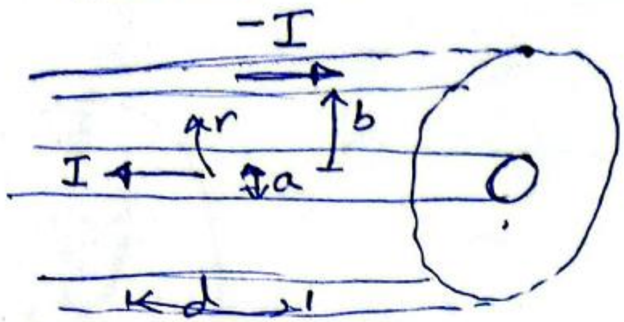
r: radius of cross section core

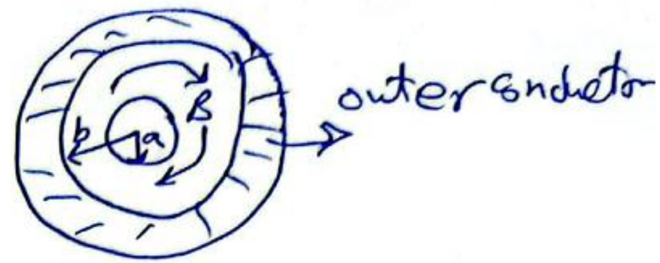
henry

R: radius of Toroid

7 – COAXIAL CABLE TL

5- Coaxial cable Transmission Line





* magnetic field is between inner & outer wire

$$\because \oint \bar{H} \cdot \bar{dL} = I_c \qquad \therefore \oint \frac{B}{\mu} \cdot \bar{dL} = I$$

$$\therefore \oint \bar{B} \cdot \bar{dL} = \mu I$$

$$B \cdot (2\pi r) = \mu I$$

B مار عليه

$$\because N = 1 \;(\text{ع الطول}) \;\&\; L = \frac{N\Phi}{I}$$

Take "d" length of wire with $N = 1$ (NO TURNS)

معندش لفات في الكابل

و ~~طول الـ~~ و $2\pi r = L$ مالفه الـ Magnetic field

$$\therefore\ B = \frac{\mu I}{2\pi r}$$

تمثل طول السلك

$$\because\ \Phi = B \cdot A \qquad \therefore\ d\Phi = B\,dA \qquad dA = d \times dr$$

تمثل سمك عنصر صغير من السلك

$$\Phi = \int B\,dA = \int_a^b B \cdot d \cdot dr$$

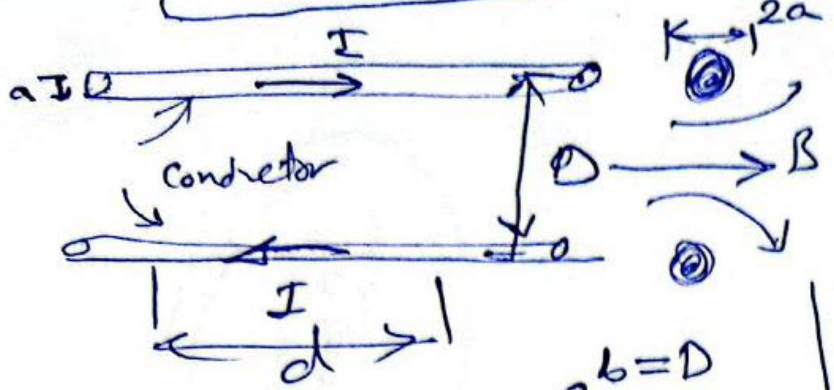
$$\therefore\ \Phi = d\int_a^b B \cdot dr = \frac{d\mu I}{2\pi}\int_a^b \frac{1}{r}\,dr$$

$$\Phi = \frac{d\mu I}{2\pi}\ln r\Big]_a^b$$

$$\therefore\ \Phi = \frac{d\mu I}{2\pi}\ln\left(\frac{b}{a}\right)$$

6- Two wire Line

لو عشان السلكين عبر بينهم تيار هيتولد
بينهم مجال وفيض فيها



- Take cross Section of Line (wire)
- line with radius a

بسبب المجال اللي بينهم مشاكل الانترنت
لو ماشي جنب سلك تليفون كابل
كهربا ضغط

$$\Phi = 2d\int_a^{b=D} B\cdot dr$$

$2d$ → Two wire

عشان كده نقطة بين السلكين

$$\Phi = \int B\cdot dA$$

$dA = 2d\cdot dr$ (two wires)

$$\therefore \boxed{\Phi = 2d\,\frac{\mu I}{2\pi}\int_a^D \frac{dr}{r} = \frac{\mu I d}{\cancel{2}\pi}\ln\frac{D}{a}}$$

$$L = \frac{N\Phi}{I}, \quad N = 1$$

$$\therefore L = \frac{\mu d}{\cancel{2}\pi}\ln D/a \text{ henry}$$

8 – TWO WIRE LINE

9 - CURL OF VECTORS

Curl of a vector

- From Ampere Law $\oint \bar{H} \cdot d\bar{L} = I = \iint_S \bar{J}_s \cdot \bar{ds}$
- From Stock Theorem $\oint_C \bar{H} \cdot d\bar{L} = \iint_S \nabla \times \bar{H} \cdot \bar{ds} = \iint_S \bar{J} \cdot \bar{ds}$

$\therefore \boxed{\nabla \times \bar{H} = \bar{J}_s}$ الصورة التفاضلية لمعادلة امبير وهي نفس معادلة ماكسويل الثانية

- For Conductor line has 2 Components

$$H_\varphi = \begin{cases} \dfrac{I}{2\pi r} & \text{outside conductor} \\ \dfrac{I r}{2\pi R^2} \rightarrow \text{inside conductor} & \text{له تكامله} \end{cases}$$

} $\nabla \times \bar{H} = \bar{J}$ داخل الـ Conductor

$\nabla \times H$ → outside conductor Because $J_s = 0$,

→ $\nabla \times H = 0$ و $J = 0$

} خارج الـ conductor

10 – MAGNETIC VECTOR POTENTIAL

10 – MAGNETIC VECTOR POTENTIAL

Magnetic vector potential

$\because \oint_s \bar{B} \cdot \bar{ds} = 0$ no charge in magnetic Field

$\rightarrow$ gauss law for magnetic field

→ use divergence theorem

$$\oint_s \bar{B} \cdot \bar{ds} = \iiint \nabla \cdot \bar{B} \, dv = 0$$

$$\therefore \boxed{\nabla \cdot \bar{B} = 0}$$

4th eq. of Maxwell

2 [illegible] مع
2 vectors

Div (curl) = 0

div (curl any vector) = 0 curl و العكس $B = \nabla \times A$

$\therefore \text{Div}(\nabla \times A) = 0$

$\because \nabla \cdot \bar{B} = 0$

$$\therefore \boxed{\nabla \times \bar{A} = \bar{B}}$$

where $\bar{A}$ called magnetic vector potential :-

it is a vector with direction the same as the direction of current causes it

$$\boxed{\bar{A} = \frac{\mu I_z l}{4\pi r} \hat{a_z}}$$

متجه في نفس اتجاه التيار ونستخدم لحساب: E، H في الـ antennas وتحديداً dipole

Thank you for your attention

Dr. Moataz Elsherbini